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TWO-DIMENSIONAL MESH ADJUSTMENT ALGORITHM

Prepared for:

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I OBJECTIVE

Many physical situations are described by functions of "boundary-layer" type with sharp variations in restricted regions. These functions are solutions of stiff, nearly singular, differential systems and are generally computed from finite difference approximations of the differential system. The truncation error of the approximation is controlled by refining the mesh in the boundary layer, where derivatives of the solution are large, while retaining as uniform a mesh as possible to raise the order of the approximation.

The objective of the research has been to establish algorithms for solution-dependent mesh adjustment.

II RESULTS

A. One-Dimensional Meshes

Several solution-dependent mesh adjustment algorithms for the one-dimensional problems presented by ordinary differential equations were obtained[1]* during the early work on the project. A mesh with points equally spaced along the graph in the (x,y) plane of the solution was found to be useful. Points equally spaced according to a measure including a proper multiple of the curvature of the graph can be very effective. As applied to a simple example, this spacing gave the same accuracy but required one-thirtieth the number of mesh points as a standard node-insertion method.

The usefulness of curvature along with distance suggested that higher derivatives of the solution should influence the mesh adjustment. When the differential equations are written as a first-order system, the phase space of the dependent variables presents the solution and all of its derivatives that influence the calculation. Equidistant nodes on the phase-space graph were shown[2] to give an error-reducing mesh.

Because many different monitor functions can be used with a given difference method of solution for a given first-order system, a "best" mesh adjustment was sought. The truncation-error minimizing (TEM) algorithm gives the most accurate solution and therefore the best mesh.

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*Numbers refer to papers in the list in Section III.

1

An equidistant mesh is found by solving the given differential system with one added, first-order, ordinary differential equation. The TEM mesh for a centered, lowest order difference scheme needs six added first-order equations. It is therefore likely to be used only occasionally as a standard by which to judge other adjustment methods. The TEM mesh was successfully computed[2] along with the solution in a simple example where the solution curve had one point of large curvature. An attempt to apply the TEM method to an example with more points of large curvature, such as the S-shaped curve of the temperature in a flow with reaction, has not yet been successful.

B. Two-Dimensional Meshes

A generalization of the equidistant mesh on the solution graph in one-dimension is an equally spaced net on the solution surface in two-dimensions. The whole mesh is determined when the four corner points of the rectangular computational domain have been located on the region boundary. Boundary mesh points are evenly spaced between the corners, and interior mesh points are then readily found by alternating direction iterations. Adjustment of the corner locations to obtain as orthogonal a mesh as possible gave successful mesh adjustments for several simple examples[3] in a circular region. The meshes were apparently distorted only as much as necessary to fit the boundary and provided the desired narrow, nearly rectangular mesh quadrilaterals in the layer of sharp variation in the solution.

Application of the same method to the supersonic flow over a chisel-shaped airfoil resulted in a very distorted mesh.[4] This distortion could be due to the presence of two layers of sharp variation at the leading shock and at the rarefaction from the shoulder of the chisel rather than the single layer of the simple examples. The distortion could also be due to the shape of the outer boundary of the flow region; the flow region corners seemed to capture the moveable computational corners and prevent the reduction in distortion seen in the simpler examples.

III PUBLICATIONS LIST

1. Ablow, C. M., and S. Schechter, "Campylotropic Coordinates," J. Computational Physics (June 1978).

2. Ablow, C. M., S. Schechter, and W. H. Zwisler, "Node Selection for Two-Point Boundary Value Problems," submitted to J. Computational Physics.

3. Ablow, C. M., and S. Schechter, "Generation of Boundary and Boundary-Layer Fitting Grids," Proceedings, Workshop on Numerical Grid Generation Techniques, Hampton, Virginia, NASA Conference Publication 2166 (October 1980).

4. Ablow, C. M., "Equidistant Mesh for Gas Dynamic Calculations," Proceedings, Symposium on the Numerical Generation of Curvilinear Coordinate Systems, Nashville, Tennessee (April 1982).

IV PERSONNEL

C. M. Ablow has been principal investigator. S. Schechter has been co-investigator during the first few years of the project with main responsibility for the numerical solution methods. W. H. Zwisler completed the initial programming in a convenient, modular, portable form.

V INTERACTIONS

The following papers were presented during the contract period:

- "Campylotropic Coordinates," paper presented at a meeting of the American Mathematical Society, Hayward, California (April 1977).

- "Node Selection for Two-Point Boundary Value Problems," paper presented at a meeting of the Society for Industrial and Applied Mathematics, Denver (November 1979).

- "Generation of Boundary and Boundary-Layer Fitting Grids," paper presented at the NASA Workshop on Numerical Grid Generation Techniques, Hampton, Virginia (October 1980).

- A Colloquium talk on the paper was presented at the Air Force Institute of Technology WPAB (October 1980).

- "Equidistant Mesh for Gas Dynamic Calculations," paper presented at the Symposium on Numerical Generation of Curvilinear Coordinate Systems, Nashville (1982). The airfoil example in the paper was suggested by J. S. Shang of FIMM/AFWAL.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Mesh adjustment by the use of solution-dependent monitor functions that take equally spaced values at mesh nodes has been studied mainly by means of examples. The simplest effective monitor is distance along the solution curve or surface. This monitor provides the desired concentration of nodes where the solution has large variation. A monitor depending on both distance and curvature can be much more effective. A "best" monitor that

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Abstract (Continued)

minimizes the truncation error of the difference method of numerical
solution has been found and used as a standard of comparison. Applica-
tion of the monitors to reactive flows or flows over an airfoil has been
impeded by as yet unexplained technical difficulties.

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